DESIGN OF HORIZONTAL PNEUMATIC TRANSPORT OF FINE-DISPERSION MATERIALS IN A DENSE MEDIUM

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Horizontal pneumatic transport in a dense medium is treated as the flow of a two-phase system with variable porosity. With the aid of Chisholm's theoretical model, it is possible to derive fundamental design equations. A coefficient is introduced which accounts for the pneumatic transportability of a material.

In selecting the input data for the optimum design of a pneumatic transport process, one assumes the availability of a sound design method which takes into account the characteristics of the route as well as the physicomechanical properties of the material to be transferred.

A method is proposed here for calculating the basic parameters of pneumatic transport, with one of the hydrodynamic models of two-phase flow taken as the tentative design scheme. The application of such schemes is founded on the specific characteristics of pneumatic transport in a dense medium [1, 2] as well as on the fact that certain empirical relations hold for the flow of a water -air mixture as much as for pneumatic transport in a dense medium [3].

We will assume that at the entrance section of a transport duct the fine-dispersion material is in a fluidized state close to the stability limit ($\epsilon \approx \epsilon_{s.l.}$) but that farther along the route there occurs a stratification of the aerosol with the solid particles precipitating in the lower part of the duct. The length of duct where this stratification occurs will depend on the physicomechanical properties of the material and can be characterized by a coefficient ξ . By analogy with the reciprocal of the relaxation time in the theory of relaxation processes, ξ may be called the reciprocal of the stratification length and may serve as a universal criterion of pneumatic transportability for a given material.

On this premise, then, horizontal pneumatic transport in a dense medium represents a flow of a two-phase stream through a duct, with the particles in a low-concentration mixture moving along the top (conventional pressurized pneumatic transport) and with the aerosol of a lengthwise variable porosity moving along the bottom:

$$\varepsilon = \varepsilon_{s,1} \{ 1 - \beta_c \left[1 - \exp\left(- \xi L_{s,1} \right) \right] \}, \tag{1}$$

where

 $\beta_0 = \frac{\varepsilon_{\text{ref}} - \varepsilon_0}{\varepsilon_{\text{s.1.}}} \,. \tag{2}$

The smaller the coefficient ξ is, therefore, the longer is the distance along the route where the fine-dispersion material remains in a quasifluidized state, the higher is its mobility, and the better are the transport conditions.

We will now use Chisholm's theoretical model and examine the equation in [4]:

$$\mu = \frac{G_{\rm M}}{G_{\rm A}} = Z \left(\frac{\rho_{\rm M}}{\rho_{\rm A}}\right)^{0.5} \left(\frac{\lambda'_Z}{\lambda'_Z}\right)^{0.5} \left(\frac{\alpha}{\beta}\right)^{0.5} \frac{1-\varphi}{\varphi}$$
(3)

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In a duct section we replace the area covered with aerosol by a fictitious sector with the vertex angle Θ (Fig. 1). In such a configuration, then, the ratios α and β of the hydraulic diameter of the material (regarded as the liquid phase) to the hydraulic diameter of the aerosol in a two-phase stream or in a one-phase stream, respectively, are:

$$\alpha = \frac{1 - \varphi}{1 - \frac{\alpha_1}{2\pi}}, \qquad (4)$$

 $\beta = \frac{2\pi\varphi}{\alpha_1} \,. \tag{4a}$

From the geometry of this arrangement follows

$$\alpha_2^4 + 12\alpha_2^2 + 24\alpha_2 = 24 \left[1 + 2\pi \left(\varphi - 0.25\right)\right],\tag{5}$$

where $\alpha_2 = [(\pi/2) - \alpha_1]$.

for calculations.

A solution of Eq. (5) on a digital computer yields a functional relation $\alpha_1 = f(\varphi)$ for the entire range of fraction φ ($0 \le \varphi \le 1$). Then

$$\frac{\alpha}{\beta} = \frac{1-\varphi}{\varphi} \frac{f(\varphi)}{2\pi - f(\varphi)} .$$
(6)

We next examine the expression which describes the interaction between phases at their boundary:

$$Z = \left[\frac{1 + \frac{S}{(1 - \varphi) \Delta P}}{1 - \frac{S}{\varphi \Delta P}} \right]^{0.5},$$
(7)

where

 $S = b\tau_s, \tag{8}$

$$b = D \sin \left[\frac{-f(\varphi)}{2} \right].$$
(9)

The following relation has been proposed in [5] for the calculation of tangential shearing stresses during pneumatic transport in a dense medium:

$$\tau_s = \lambda_s \mu_b \rho_A (v_A')^2 \left(1 - \frac{v_M}{v_A'} \right).$$
(10)

Assuming, as in [6]

$$v'_{\rm M} = 2\bar{v}_{\rm M},\tag{11}$$

we have after a few transformations

$$S = 4\lambda_{s}n\mu D \frac{\rho_{\rm A}}{g} \frac{1 - k_{1}}{k_{1}^{2}} \frac{G_{\rm M}^{2} [1 - \exp{(-\xi L_{\rm ref})}]}{\rho_{\rm M}^{2} (1 - \varphi)^{2} A^{2}} \sin\left[\frac{f(\varphi)}{2}\right],$$
(12)

where $k_i = v'_M / v'_A$ and $n = \mu_b / \mu$.

The density of aerosol in the lower part of the duct is

$$\rho_{\rm M} = \rho_{\rm M}' \left\{ 1 - \varepsilon_{\rm s,1} \left[1 - \beta_0 \left(1 - \exp\left(-\xi L_{\rm ref.} \right) \right) \right] \right\}$$
(13)

and the ratio of densities of the phases, during isothermal expansion of air is

$$\frac{\rho_{\rm M}}{\rho_{\rm A}} = \frac{\rho_{\rm M}' P_0}{\rho_{\rm A}' (P_0 + \Delta P)} \left\{ 1 - \varepsilon_{\rm ref} \left[1 - \beta_0 \left(1 - \exp\left(- \xi P_{\rm S,1} \right) \right) \right] \right\}. \tag{14}$$

Based on this model of two-phase flow, the friction coefficient for particles of the material in the upper part of the duct is $\lambda'_Z \approx 0.005$.

The friction coefficient for the material in the lower part of the duct is

$$\lambda_{z}^{*} = \lambda_{z}^{*} \left[1 - \exp\left(-\xi L_{\text{ref}}\right) \right] + \lambda_{z}^{*}$$
(15)

We will also use the relation in [2] for calculating the total pressure drop ΔP :

$$\Delta P = \lambda^* \frac{\omega_0^2}{2g} \rho'_{\rm M} \left(1 - \varphi_0\right) (1 - \varepsilon) \frac{L_{\rm ref}}{D} \left(\frac{d}{D}\right)^{0.7}.$$
 (16)

Substituting for the quantities in (3) their values from (6), (12), (14), (15), and (16), we finally obtain the following transcendental equation:

$$\mu = \left\{ \left[\lambda^{*} k_{1}^{2} \left(\rho_{M}^{'} \right)^{3} V^{2} \left(1 - \varepsilon \right) \varphi \left(1 - \varphi \right)^{2} \left(1 - \varphi_{u} \right)^{2} + \lambda_{s} \frac{32}{\pi} \left[1 - \exp \left(- \xi L_{ref}^{'} \right) \right] n \mu^{3} \left(1 - k_{1} \right) \left(\rho_{u}^{'} \right)^{3} \left(1 + \frac{\Delta P}{P_{u}} \right)^{3} \right] \right\} \\ \times \sin \left[\frac{f(\varphi)}{2} \right] \left(\frac{d}{D} \right)^{0.7} \frac{\varphi}{1 - \varphi} \right] \left(\lambda^{*} k_{1}^{2} \left(\rho_{M}^{'} \right)^{3} V^{2} \left(1 - \varepsilon \right) \varphi \left(1 - \varphi \right)^{2} \left(1 - \varphi_{u}^{0} \right)^{2} \right) \\ - \lambda_{s} \frac{32}{\pi} \left[1 - \exp \left(- \xi L_{ref} \right) \right] n \mu^{3} \left(1 - k_{1} \right) \left(\rho_{A}^{'} \right)^{3} \left(1 + \frac{\Delta P}{P_{u}} \right)^{3} \right) \\ \times \sin \left[\frac{f(\varphi)}{2} \right] \left(\frac{d}{D} \right)^{0.7} \right] \right\}^{0.5} \left[\frac{\rho_{M}^{'} P_{u}}{\left(P_{u} + \Delta P \right) \rho_{A}^{'}} V \right]^{0.5} \\ \times \left\{ \frac{\lambda_{z}^{*}}{\lambda_{z}^{'}} \left[1 - \exp \left(- \xi L_{ref} \right) \right] + 1 \right\}^{-0.5} \left[\frac{1 - \varphi}{\varphi} \frac{f(\varphi)}{2\pi - f(\varphi)} \right]^{0.5} \frac{1 - \varphi}{\varphi} ,$$

where $V = 1 - \varepsilon_{s.l.} \{ 1 - \beta_0 [1 - \exp(-\xi L_{ref})] \}.$

Equation (17) was solved on a digital computer, to find the relative weight concentration u at various lengths of the transport route L_{ref} and various coefficients of friction between particles and the duct wall λ_Z^* , also the universal pneumatic transportability ξ was determined first for one of the experiments with a known concentration μ .

The $\mu = f(L_{ref})$ curves shown in Fig. 2 have been calculated for various values of coefficient ξ . In Fig. 3 are shown $u = f(L_{ref})$ curves which have been obtained by solving Eq. (17) for $\xi = 0.1$ and $\varphi_0 = 0.2$ as well as test curves from [9].

Based on the satisfactory agreement found here, one may conclude that these relations are suitable for the analysis and the design of horizontal pneumatic transport in a dense medium. Furthermore, we can determine the slip coefficient at the entrance section of a duct:

$$k = \frac{1 - \varphi_0}{\varphi_0} \cdot \frac{\rho_{\rm M} \left(1 - \varepsilon_{\rm S,1}\right) P_0}{\rho_{\rm A}'(P_0 - \Delta P) \mu}$$

The actual weight concentration of the mixture, taking into account the extra air discharge into the air chamber to replace the material [3], is determined according to the formula

$$\frac{\mu_{ac}}{1 + \mu \frac{\dot{\rho}_{A}}{\rho_{M}} \left(1 + \frac{\Delta P}{P_{0}}\right)}$$

It is to be noted that the fill factor at the entrance (φ_0) depends on the design of the air chamber and on the method of feeding the air for transport [10]. When the referred air velocity (ω_0) is given, then the unloading rate will be

$$G_{\rm M} = \rho_{\rm M} (1 - \varphi_0)(1 - \varepsilon_{\rm s,1}) \omega_0 k^{-1} A$$

and the air-flow rate

 $G'_{\mathbf{A}} = G_{\mathbf{M}}/\mu_{\mathbf{ac}}$.



Fig. 2. Weight concentration of the mixture μ (kg/kg) as a function of the referred duct length L_{ref} (m), at various values of coefficient ξ : 1) 0.01; 2) 0.05; 3) 0.1; 4) 1.0; 5) 2.0; friction coefficient $\lambda_Z^* = 0.6$, referred air velocity $\omega_0 = 3$ m/sec.

Fig. 3. Comparison between calculated $\mu = f(L_{ref})$ curves: 1) $\omega_0 = 3$ m /sec; 2) $\omega_0 = 5$ m/sec; 3) $\omega_0 = 7$ m/sec, with $\lambda_Z^* = 0.5$ and $\xi = 0.01$, and tested relations (curves 4 and 5) according to [9].

With the aid of these formulas, one can design an optimum pneumatic transport process and ensure operation of the apparatus with a minimum waste of energy.

NOTATION

d	is the diameter of a particle, m;
D	is the duct diameter, m;
A	is the duct area, m^2 ;
$L_{ref} = \alpha'(L_e + L)$	is the referred duct length, m;
Le	is the duct length equivalent to local resistances, m;
$\alpha' = 0.15$	is a coefficient;
₀ ع	is the porosity of the bed;
^ε s.1	is the porosity at the stability limit;
ε	is the instantaneous porosity;
ξ	is the reciprocal of the stratification coefficient;
GM	is the transport productivity, kg/sec;
GA	is the air flow rate, kg/sec;
μ	is the concentration in the mixture, kg/kg;
$\mu_{\rm b}$	is the boundary concentration in the upper zone of a duct, kg/kg;
$\lambda'_{\mathbf{Z}}$	is the friction coefficient for aerosol flow in the upper part of a duct;
$\lambda_{\rm Z}^{\rm m}$	is the friction coefficient for aerosol flow in the lower part of a duct;
$\lambda_{\mathbf{Z}}^{*}$	is the friction coefficient for flow of solid particles in a duct;
λ_{s}^{-}	is the referred friction coefficient for flow along the interphase boundary;
λ*	is the referred friction coefficient for calculating the total pressure drop ΔP ;
$\rho_{\mathbf{M}}$	is the density of the material at the interphase boundary, kg/m^3 ;
$\rho_{\mathbf{A}}^{\prime}$	is the density of air at the interphase boundary under atmospheric pressure P_0 , kg
**	$/m^{3};$
v _M	is the velocity of the material at the interphase boundary, m/sec ;
<u>v'</u> A	is the air velocity at the interphase boundary, m/sec;
vM	is the mean velocity of the material, m/sec;
φ	is the fraction of the duct cross section covered with aerosol;
τ_s	is the tangential shearing stress at the interphase boundary, kg/cm^2 .

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